This book deals with the differential geometry of surfaces from the point of view of singularity theory. It presents a thorough survey of the theory of contact betweeen manifolds and its link with the theory of caustics and wavefonts. The powerful techniques of these theories are used to deduce geometric information about surfaces immersed in the Euclidean 3, 4 and 5-spaces as well as spacelike surfaces in the Minkowski space-time.

The book consists of ten chapters.

Chapter introduces basic differential geometric notions and shows how singular theory can be used to recover classical results on curves and surfaces in a simple and more elegant way. They also highlight how singular theory reveals the rich and deep underlying concepts involved. First, classical differential geometry techniques are used to obtain the shape of the evolute and parallels of a plane curve. The distance squared functions on the plane curve is defined and the geometric information about the curve is recovered from the singularities type of the members of this family. The Lagrangian and Legendrian singularity theory framework is used to deduce properties of the evolute that are invariant under diffeomorphisms. The case of surfaces in the Euclidean 3-space and the singularities of their focal sets is considered similarly. The last section is devoted to the singularities of ruled and developable surfaces.

Chapter 2 introduces basic facts about the extrinsic geometry of submanifolds of Euclidean spaces. In particular, totally umbilic hypersurfaces are presented. The extrinsic geometry of a general hypersurfaces in $\mathbb{R}^{n+1}$ is studied in subsequent chapters by looking at its contact with totally umbilic hypersurfaces.

Chapter 3 presents basic definitions realted with singularities of smooth mapppings and states the results of finite determinacy and versal unfoldings. Those results are fundamental in the study of the geometry families of mappings on surfaces treated in the book.

Chapter 4 recalls the contact theory, theory introduced Mather and developed by Montaldi. In particular, the authors consider the notion of contact between manifolds and Thom's transversality theorem as main tools for investigating generic properties of submanifolds of $\mathbb{R}^n$.

In chapter 5, the authors consider the theory of contact from the view point of the Lagrangian and Legendrian singularities. First, they recall some basic concepts in symplectic and contact geometries and then they establish the fundamental result that links the theory of contact with that of Lagrangian and Legendrian singularities.

In the next four chapters the singular theory techniques are used to study a submanifold M of Euclidean spaces and of Minkowski space-time locally at a given point on M. In particular, chapter 6, 7 and 8, are devoted to study the extrinsic differential geometry of surfaces embedded in the Euclidean space 3, 4 and 5-spaces respectively, and chapter 9 is devoted to the study of spacelike surfaces in 4-dimensional Minkowski space-time (surfaces in Euclidean 3-space and surfaces in hyperbolic space are special cases).

Finally, in chapter 10 the authors present some applications of singular theory to the study of global properties of submanifold. The approaches considered in this chapter are the following: The first approach uses a stratification of the parameters space of the family of functions and mappings defined on M, the second is a topological approach, and the third one considers the Poincaré-Hopf formula.

Although the present book, as the authors explain in the preface, is focused on their research results and on their own and collaborators interests, it has been enriched by including, in the Notes of each chapter, other aspects and studies on the topics in questions and by providing a widely list of references.

The book will be a helpful tool for researchers interested in the filed and in particular, in the study of the differential geometry of singular submanifolds (such as caustics, wavefronts, etc.) of Euclidean and Minkowski spaces.