The Mumford-Tate groups and Domains

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Mumford-Tate groups and domains live in the boundary of several branches of mathematics. As Hodge structures, these objects are endowed with several information codified in geometrical and arithmetical language. Furthermore, they are groups, adding more information and properties. The authors are aware of such confluence, and clearly the entire work points out to this: in each chapter of this contribution one of these three aspects prevails. This makes more valuable the results presented in this book because the biggest and most interesting theorems arise from the interplay of differents "flavours" of mathematics and, in this way, the present book becomes a landmark in this subject.

As it happens in several books on very technical topics, the authors assume a considerable amount of background. In this case, readers must have a deep knowledge in Lie groups and algebras (especially in Chapter IV), aside from some familiarity with related topics in Mumford-Tate groups and period domain (such as Hodge theory, deformation theory, etc.). Nevertheless, the three first chapters serve as an introduction to the key issues to be fully treated in the following ones and they furnish a dense survey on the so-considered basic properties and the subtle relations which hold all these objects. Although the topics covered in these pages is fairly difficult, both the treatment and the good exposition style make the argument easy to follow.

In Chapter IV, containing the main part, the authors try to answer the natural question "What are the possible Mumford-Tate groups?" which is a reformulated classical question posed in earlier works. Here, the use of Lie algebras and groups gives surprising characterizations which translate such a question to an equivalent one in Lie algebras. These results are applied to obtain a complete list of real forms with Hodge representations. Chapter V is devoted to characterize the CM Hodge structures (a Hodge structure with complex multiplication) in terms of its Mumford-Tate group.

Chapter VI is devoted to the arithmetic aspects of the Mumford-Tate domains and their relation to the Noether-Lefschetz loci. On the other hand, Chapter VII develops an algorithm in order to determine all Mumford-Tate subdomains of a given period domain; such results are applied to obtain a complete classification of CM Hodge structures in certain ranks and weights. The book finishes discussing some arithmetical aspects presented in parametrized families of smooth projective varieties in connection to the Noether-Lefschetz loci.

The brilliance of the results and their broad spectrum of their applications makes this book an outstanding piece. Yet, there is more to write and to develop: the authors suggest the existence of future lines of research for a next book.

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