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Stochastic Tools in Mathematics and Science
(Third Edition)
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The present book, published in 2013, is the third edition of the book first published in 2005 and with a second edition in 2009. In the preface the authors say: With the experience of having taught several courses on the subject (at the University of California at Berkeley), since publication of the previous editions we have reorganized the contents to clarify the connections between the different topics, making it easier to read, and to lead the reader to the environment of ongoing research on the issues.

The core of the book consists on the study of stochastic processes and their applications. Along the first three chapters they are introduced in a brief, but clear and precise way, the mathematical subjects necessary for the study of stochastic processes: general properties of pre-Hilbert and Hilbert spaces, Fourier series, Fourier transform, probability, mathematical expectation and moments of random variables, conditional expectation, central limit theorem (theorem Lindeberg-Levy), Monte Carlo methods that allow to evaluate a non-random number by the expectation of a random variable, estimation and Bayesian estimation.

The next three chapters deal with stochastic processes. It begins with the study of a particular stochastic process, namely the Brownian motion or Wiener process, which is fundamental to applications in many branches of Science in particular to statistical mechanics and the financial economy (subject not treated in the book). It is studied in relation with the heat equation including the heat equation with potential, which is solved by the Feynman-Kac formula. In the same vein, branching Brownian motion is used to solve the Kolmogorov-Petrovskii-Piskunov nonlinear differential equation.
The stochastic integral with respect to a Wiener process is introduced from the Wiener measure in the space of continuous maps over $[0, 1]$ fixing the origin, and the $\sigma-$algebra generated by the cylinders of this space. The integral of the functionals over this space relative to this measure is called Wiener integral. The physicists version of the Wiener integral as path integral is established and the integration technique is presented by the series expansion of a perturbation, being emphasized that for this technique it is useful the graphical representation of the different terms in a diagram called Feynman diagram (procedure analogous to the Feynman integrals). The

generalizations of the Wiener integral due to Itô, on the one hand, and to Fisk-Stratonovich, on the other, are briefly indicated and by an example it is shown that the two integrals are different.
The stochastic differential equations considered are those of the form $du = a(t, u(t))dt + f(t)dW$, where W is a Wiener process. From this type of stochastic differential equations the Langevin equation (important in applications) $du = -au(t)dt + dW$, is studied in detail. From this, it is obtained the Fokker-Planck equation or Kolmogorov forward equation, that describes the evolution of the transition density of the Markov stochastic process solution of the Langevin equation.
Stationary stochastic processes in the wide sense and in the strict sense are discussed, and the theorem of Khinchin (referred in other books as Bochner-Khinchine theorem) is established, from which it follows that if a stationary stochastic process in the wide sense has spectral density, then its covariance function is the Fourier transform of the spectral density. These results are applied to the study of turbulence in a fluid.

In the last three chapters some applications to statistical mechanics are considered. The Lagrangian and Hamiltonian formalism of classical mechanics are recalled, allowing to present the basic ideas of statistical mechanics and to establish the Liouville equation. The essential concepts of statistical mechanics (entropy, temperature, equipartition, ergodicity) are introduced and studied and the Ising model (model of ferromagnetism which provides the simplest example of a system in thermal equilibrium) is presented in a simplified manner, clarifying the concepts introduced (eg, computational Monte Carlo technique in statistical mechanics of ergodic Markov chains). For the case of a system not in thermal equilibrium, it is introduced the Mori-Zwanzig formalism, which permits to reduce the number of variables in the equations governing the system.

At the end of each chapter there is a collection of exercises that extend important aspects of the topics covered in that chapter or introduce new theoretical issues, such as the characteristic function of a random variable, which appears in the exercises in Chapter 6.
The bibliography is detailed by chapters.
Finally, the edition of the book is very well cared and only a few easily correctable errors by the reader can be observed (ie. On page 22, line 12, "v" should be replaced by "k". In page 30, line -6, "variable" should be replaced by "variance" and in the same page, line -5, "variance" should be replaced by "variable". In pages 66 and 67, reference (3.1) must be replaced by (4.1).)

The book is not a systematic treatise on the theory of stochastic processes but, with the topics chosen, the authors present in only 200 pages, and in a clear and precise way, some of the most relevant aspects of the theory of stochastic processes and their applications. This makes this book an excellent introductory book to the study of these issues.

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