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CONCRETE FUNCTIONAL CALCULUS

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The present monograph is quite extensive and interesting. It is divided
into twelve chapters on different topics on Functional calculus and an ap-
pendix on non-atomic measure spaces. The monograph is devoted to several
smoothness problems for concrete nonlinear operators acting in "concrete"
Banach function spaces. The role of the variability of vector valued functions,
namely the bounded p-variation and more general the bounded Φ-variation
(for Φ being an Orlicz function) is deeply analyzed in relation to several
important topics.

The monograph starts with a careful study of several extensions of the
classical scalar Riemann-Stieltjes Integral and Kolmogorof integral to the
functions valued in Banach lattices, Banach spaces and Banach algebras (for
example integrals of Young-Stieltjes, Henstock-Kurzweill, Ward-Perron,...).
Associated integral inequalities like the Love-Young inequality are studied
for these vector integral extensions. The book continues studying the prop-
erties of the Two Function Composition operator i.e. the composition map
$TC : (F,G) \to F \diamond G$. The mapping properties of the TC operator are deter-
mined by the variability of the functions F and G. As an important partic-
ular case for a fixed function F we have $Nemyttskii$ Operators (which have
been considered for instance in the monograph of J.Apell and P.B.Zabrejko:
"Non linear superposition operators" Cambridge Tracts in Mathematics 96
Cambridge University Press).

Chapters 6 and 7 of the book are dedicated to the differentiability of
these Nemyttskii operators. In the two previous chapters the general facts
on Banach algebras and the differentiability in Banach spaces had been in-
troduced. This helps a lot in reading subsequent chapters since many needed
concepts and results are included previously.

Other interesting topics considered in the monograph concerning func-
tions of bounded Φ-variation are Integral Products, Fourier Series (the or-
der of decreasing of Fourier coefficients,...), and non-lineal integral equa-
tions. The final extensive chapter is devoted to Stochastic Processes, study-
ing Markov processes and Ito integrals as well as asymptotic properties of

empirical processes.

The book has many historical comments and remarks which clarify the developments of the theory. It has also an extensive bibliography with 258 references.

In conclusion this is a very interesting and extensive book (671 pages!) which will be very useful for all interested readers in Real-Functional Analysis and Probability.

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