A common theme in algebra is to understand algebraic structures over arbitrary fields by first studying them over their algebraic closure and then investigating the possible ways to "descend" to the base field again. A typical example occurs in the theory of quadratic forms over an arbitrary field. In order to decide when two given quadratic forms are non-isometric, a useful tool is to define invariants for quadratic forms; typical (easy) examples of such invariants are the discriminant and the Clifford algebra. These are examples of cohomological invariants (of degree 1 and 2, respectively).

Our goal is to study cohomological invariants for non-associative algebras and their related linear algebraic groups. Examples that have been well studied are octonion algebras (certain 8-dimensional algebras) and Albert algebras (certain 27-dimensional Jordan algebras); these are connected to groups of type G2 and F4, respectively. We will study invariants of structurable algebras, a class of algebras with involution, simultaneously generalizing Jordan algebras and associative algebras with involution. We will mainly be interested in the exceptional structurable algebras: the 35-dimensional Smirnov algebras; tensor products of two composition algebras (of dimension 16, 32, 64); structurable algebras of skew-dimension one arising from hermitian cubic norm structures (of dimension 8, 20, 32, 56). These examples are related to groups of type 3D4, E6, E7 and E8.
### FUNDING PER HOST INSTITUTION

**Ghent University**

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### STAFF

**Ghent University**

**Staff type**  
Scientist - fulltime

**Motivation**  
An essential part of the proposed research will be carried out by a (fulltime) PhD student.

**Name**  
//

**Academic degree**  

**Current employer**
### CONSUMABLES

**Ghent University**

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### DISCIPLINES

Algebra
RESEARCH CONTEXT

How this project fits in the research activities of the research group

*If the project has already been initiated, please state the progression of your research.*

Our research group has built a strong experience in the theory of structurable algebras, which is the main theme of the proposed project. In particular, I have written several publications in this area, in joint work with Lien Boelaert, a former member of the research group, and all of which appeared (or will appear) in important journals (Transactions of the AMS, Proceedings of the LMS, Memoirs of the AMS); see references [BD13], [BD15] and [BDS17] in the bibliography. Another result in this area that I obtained in independent work, is an explicit description of the structurable algebras of skew-dimension one; this was mentioned as one of the important open problems in Allison's Oberwolfach proceedings of a conference held in August 1992, which remained unsolved until now.

It is also relevant to mention that there is a pending application for an FWO PhD fellowship (by Jeroen Meulewaeter) which has a related research topic, namely the connection between structurable algebras and incidence geometries through representation theory of linear algebraic groups. There is no danger of overlap with the current project because the goals are completely different, but at the same time, it is of course interesting that there is a common theme, and this will prove valuable during our meetings, seminars, etc. (Our research group currently has such a seminar about once every two weeks, and more than once, the discussions arising during such a seminar have proved effective and resulted in new ideas.)

National and international context

The long paper "Moufang sets and structurable division algebras" (L. Boelaert, T. De Medts, A. Stavrova, to appear in the Memoirs of the AMS, 100 pages) was the result of a longstanding project in collaboration with Anastasia Stavrova from Sint-Petersburg (Russia) that was spread out over several years. This was an excellent example of an initially unexpected collaboration that eventually culminated in results where the different competences of the two research groups were both equally crucial.

We initially learned about structurable algebras from Skip Garibaldi --who recently left the academic world and took a position at the Air Force Scientific Advisory Board of the CCR-- through regular contacts, both by email and by research visits (at that time at Emory University in Atlanta, USA).

Collaboration

Not applicable, as there is only one research group and one supervisor involved in this project.

EXTRA DATA

Funding applied for elsewhere or already available

Not applicable.
In the table below questions are listed on the ethical aspects of your research proposal.

If you mark a ‘yes’ for the question, it follows that

- **For the questions marked with *:** the applicant is legally or on the basis of institutional regulations obliged to ask for an ethical advice at the competent ethics committee of the host institution; please do take into account that even when there is no obligation with regard to the research itself, for the publication of the results a positive advise still can prove to be necessary.

  If you have answered questions with a * positively, you must submit your proposal to the ethics committee **as soon as your application has been approved for funding**. Your project can only start when this clearance has been formally given. Only if the advice relates to a work package that is planned for a later stage of the project, it may be submitted just before the start of that part of the research. Please keep in mind that the advisory procedure can take some time and that therefore you should submit your proposal to the ethics committee well in time.

- **For the questions that are not marked:** the applicant and the evaluation panel are invited to reflect on the issue and take, if necessary, the necessary precautionary measures.

  You find more on the FWO policy and procedure concerning ethical issues and on legal and other documents on the **FWO web page dedicated to that topic**.

I confirm that none of the issues below apply to my proposal. True

I hereby confirm having taken note that an ethical clearance is needed for the start of my project. I will thus ensure submission of my proposal to the research ethics committee of my host institution.

Please specify which ethics committee(s) deal(s)/will deal with your application.

In case you will submit your proposal to the committee only before the start of work package(s) (WP) that are concerned:

<table>
<thead>
<tr>
<th>Number/description of WP(s)</th>
<th>Starting date of WP(s)</th>
</tr>
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</table>

Applicant: Tom De Medts | Application number:
1. Human Embryos/Foetuses

ETHICS ADVICE RELATED TO THESE QUESTIONS SHOULD ALWAYS BE REQUESTED BEFORE THE START OF THE RESEARCH PROJECT AS A WHOLE AND ALSO REQUIRE AN EXAMINATION BY THE FEDERAL COMMISSION FOR EMBRYOS

Does your research involve Human Embryonic Stem Cells (hESCs)?*  
- Will the hESCs be directly derived from embryos within this project?  
- Are the hESCs previously established cell lines?  

Does your research involve the use of human embryos?*  

Does your research involve the use of human foetal tissues / cells?*  

2. Humans

Does your research involve human participants?*  
- Are they volunteers for social or human sciences research?  
- Are they persons unable to give informed consent?  
- Are they vulnerable individuals or groups?  
- Are they children/minors?  
- Are they patients?  
- Are they healthy volunteers for medical studies?  

Does your research involve physical interventions on the study participants?*  
- Does it involve invasive techniques?  
- Does it involve collection of biological samples?  

3. Human Cells/Tissues

Does your research involve human cells or tissues (other than from Human Embryos/Foetuses, i.e. section 1)?*  
- Are they obtained from commercial sources?  
- Do they originate from another laboratory/institution/biobank?  
- Were they produced or collected by you from previous research activities?  
- Are they produced or collected by you as part of this project?²  

4. Personal Data

Does your research involve personal data collection and/or processing?* (¹)  
- Does it involve the collection and/or processing of sensitive personal data?  
- Does it involve collecting/processing of genetic information/data?  
- Does it involve tracking or observation of participants?  

Does your research involve further processing of previously collected personal data (‘secondary use’)?*
5. Animals

Does your research involve research procedures to live non-human vertebrate animals (incl. independently feeding larval forms, foetal forms of mammals in the last trimester of their normal development and cephalopods, and also forms in earlier stages if the experiments have consequences in later stages)?

- Are they vertebrates or live cephalopods? N/A
- Are they non-human primates? (²) N/A
- Are they genetically modified animals? N/A
- Are they cloned farm animals? N/A
- Are they endangered species? N/A

6. International Collaboration

Do you plan to use local resources (e.g. animal and/or human tissue samples, genetic material, live animals, human remains, materials of historical value, endangered fauna or flora samples, etc.)? N/A

Do you plan to import/export any material from/to other countries? N/A

Name of country/ies:

If your research involves low and/or lower middle income countries, are benefits-sharing measures foreseen? N/A

Could the situation in the country put the individuals taking part in the research at risk? N/A

7. Environment & Health and Safety

Does your research involve the use of elements that may cause harm to the environment, to animals or plants? N/A

Does your research deal with endangered fauna and/or flora and/or protected areas? N/A

Does your research involve the use of elements that may cause harm to humans, including research staff? N/A

8. Dual Use

Does your research have the potential for military applications? N/A

9. Misuse

Does your research have the potential for malevolent/criminal/terrorist abuse? N/A

10. Other Ethics Issues

Are there any other ethics issues that should be taken into consideration? Please specify.
For these issues the Belgian commission on privacy protection (Commissie voor de bescherming van de persoonlijke levenssfeer) has to be consulted. You cannot consult the commission directly, but always first contact the research coordination of your host institution.

In this case you already have to submit your proposal to the ethics committee in the application phase.
Indicate the state of the art.
A common theme in algebra is to understand algebraic structures over arbitrary fields \( k \) by first studying them over their algebraic closure \( K \) and then investigating the possible ways to "descend" to the base field \( k \) again. This theme is, in fact, well known even in other areas: objects over the real numbers can sometimes be understood by first studying them over the complex numbers and then descending back to the reals, for instance in the theory of Lie groups, where the objects of interest are so-called real forms of complex Lie groups.

A typical algebraic example occurs in the theory of quadratic forms over an arbitrary field \( k \). Over the algebraic closure \( K \), such a quadratic form is completely determined (up to isometry) by its dimension. This is not true at all over arbitrary fields, and already the study of quadratic forms over the field \( \mathbb{Q} \) of rational numbers, for instance, is highly non-trivial. An inevitable question is whether two given quadratic forms of the same dimension are non-isometric, and this leads naturally to the theory of invariants, i.e., certain "properties" that are invariant under isometry. (In fact, the dimension of the quadratic form is itself an example of such an invariant.)

The oldest interesting invariant of quadratic forms is undoubtedly the discriminant \( \text{disc}(q) \) of a (non-degenerate) quadratic form \( q \) over a field of characteristic not equal to 2, which we will interpret as an element of \( k^×/(k^×)^2 \). Another well known classical invariant is the isomorphism class of the Clifford algebra \( C(q) \) of \( q \), and the related Hasse invariant \( c(q) \), which is, up to a sign, the class of \( C(q) \) in (the 2-torsion part of) the Brauer group \( \text{Br}(k) \) of \( k \). As it turns out, these are examples of cohomological invariants, i.e. invariants taking values in some abelian Galois cohomology groups. In this particular case, \( \text{disc}(q) \in H^1(k, \mathbb{Z}/2\mathbb{Z}) \) and \( c(q) \in H^2(k, \mathbb{Z}/2\mathbb{Z}) \); such invariants are called mod 2 invariants. An example of higher mod 2 invariants is provided by the maps \( e_n \) that sends each \( n \)-Pfister quadratic form \( (1, −a_1) \otimes \cdots \otimes (1, −a_n) \) to the class \((a_1) \cup \cdots \cup (a_n) \) in \( H^n(k, \mathbb{Z}/2\mathbb{Z}) \). (For \( n = 3 \), this invariant is known as the Arason invariant.) Observe that in fact, the invariant \( e_n \) is defined only on the kernel of the previous invariant \( e_{n−1} \) and not on all quadratic forms, so this can be interpreted as a sequence of invariants that are "probed" subsequently until a non-zero invariant is found. The famous Milnor Conjecture, which has been proven by Voevodsky (for which he has been awarded the Fields Medal), states that each map \( e_n \) induces an isomorphism \( I^n/I^{n+1} \rightarrow H^n(k, \mathbb{Z}/2\mathbb{Z}) \), where \( I^n \) is the \( n \)-th power of the ideal \( I \) of even-dimensional forms in the Witt ring of \( k \). Since \( \bigcap_n I^n = 0 \), this means that a non-degenerate quadratic form can be completely recognized by subsequently probing the invariants \( e_n \).

More generally, we can associate cohomological invariants to linear algebraic groups. More precisely, if \( G \) is a smooth linear algebraic group over \( k \), then a cohomological invariant of \( G \) is a map \( f : H^1(k, G) \rightarrow H^n(k, \mathbb{Z}/d\mathbb{Z}) \) for some \( d \) (or for some other target group). (In fact, a cohomological invariant can best be viewed as a natural transformation between functors from the category of field extensions of \( k \) to the category of sets, i.e. \( f : H^1(∗, G) \rightarrow H^n(∗, \mathbb{Z}/d\mathbb{Z}) \), but we will avoid this formality in this project description.) For the linear algebraic group \( G = O_n \) which is smooth if \( \text{char}(k) \neq 2 \), the cohomology set \( H^1(k, O_n) = H^1(k, O_n) \) classifies
isomorphism classes of non-degenerate $n$-dimensional quadratic forms over $k$, and this is precisely the situation we have been describing in the previous paragraphs.

A particularly interesting invariant is the Rost invariant for a quasisimple simply connected linear algebraic group $G$, which is a map $r_G : H^2(k, G) \rightarrow H^3(k, \mathbb{Q}/\mathbb{Z}(2))$. (The target group $\mathbb{Q}/\mathbb{Z}(2)$ is essentially the direct limit of the groups $\mu_{2^i}$, with some subtleties when $\text{char}(k) = p > 0$.) This invariant was introduced by Markus Rost, first for groups of type $F_4$ [Ros91], and later, in unpublished work, for general simply connected groups; its existence had been predicted by J.-P. Serre almost simultaneously, and it is also Serre who first wrote it down [Ser95]. For this reason, this invariant is sometimes called the Serre-Rost invariant. An extensive study of this invariant can be found in the second part of [GMS03] (by Alexander Merkurjev); a shorter discussion can also be found in [KMRT98, §31]. Recently, this has been extended to general semisimple linear algebraic groups over arbitrary fields [Mer16b] and to reductive groups over algebraically closed fields [LM16]. Another recent improvement, in particular getting rid of some of the assumptions on the characteristic, is [GM17].

The main topic of the proposed research project will be the study of cohomological invariants of certain algebras. For associative (central simple) algebras, this has been an active research topic for some time, and we refer to work of S. Baek and A. Merkurjev for a recent account [BM09, Mer16]. For central simple algebras with involution, the investigation of cohomological invariants has been initiated by Anne Quéguiner [Qué97], who studied the discriminant and the Hasse invariant for such algebras. An excellent more recent overview on this topic is due to Jean-Pierre Tignol [Tig10].

Our focus will be on non-associative algebras, in particular those associated to exceptional linear algebraic groups. The easiest example of such a situation is the case of groups of type $G_2$, which are the automorphism groups of octonion algebras, certain 8-dimensional non-associative algebras. Since the isomorphism class of an octonion algebra is completely determined by the isometry class of its norm form (which is a 3-fold Pfister quadratic form), the investigation of cohomological invariants of such an algebra reduces to the study of cohomological invariants of these quadratic forms, and this is well understood. In particular, the Rost invariant of groups of type $G_2$ corresponds precisely to the Arason invariant of this norm form.

Much more interesting already are groups of type $F_4$, which are precisely the automorphism groups of Albert algebras, 27-dimensional exceptional Jordan algebras. These algebras admit three interesting cohomological invariants:

$$g_3 : H^1(k, F_4) \rightarrow H^3(k, \mathbb{Z}/3\mathbb{Z}),$$

$$f_3 : H^1(k, F_4) \rightarrow H^3(k, \mathbb{Z}/2\mathbb{Z}),$$

$$f_5 : H^1(k, F_4) \rightarrow H^5(k, \mathbb{Z}/2\mathbb{Z}).$$

The invariant $g_3$ corresponds to the Rost invariant, and the invariants $f_3$ and $f_5$ can be interpreted as the $e_3$- and $e_5$-invariants of a suitable 3-Pfister form $q_3$ and a 5-Pfister form $q_5$, uniquely determined by the Albert algebra $A$ by the property $q_A \perp q_3 \cong (2, 2, 2) \perp q_5$, where $q_A(x) = \frac{x^4}{2}$.
\[ \text{Tr}(x^2)/2, \] where \( \text{Tr} \) is the trace of the Albert algebra. A more direct description of the mod 2 invariants \( f_3 \) and \( f_5 \) has been provided by Holger Petersson and Michel Racine [PR95], and in a subsequent paper [PR96], they also described an elementary approach to the Rost invariant \( g_3 \) directly in terms of the Albert algebra. Yet another description of \( f_3 \) and \( f_5 \) in terms of structure constants has been found more recently by Vladimir Chernousov [Che10, section 6]. A very important open problem in the theory of Albert algebras is to determine whether these three invariants together classify Albert algebras [Gar09, Open Problem 8.7].

For the remaining exceptional groups, i.e. those of type \( E_6, E_7 \) and \( E_8 \), substantial work on their cohomological invariants has been undertaken by Skip Garibaldi in [Gar09, Part III]. The main technique is to find subgroups \( N \) of \( G \) such that the map \( H^1(k, N) \rightarrow H^1(k, G) \) is surjective. An important source of such subgroups are the point stabilizers of points in an open \( G \)-orbit in \( \mathbb{P}(V) \) for some representation \( V \), and this is used to construct invariants for groups of type \( E_6 \) and \( E_7 \). This is not applicable to \( E_8 \), however, because no representation for \( E_8 \) admits an open orbit in \( \mathbb{P}(V) \), and in this case, Garibaldi uses different ideas. Nevertheless, fundamental open questions about the invariants of \( E_7 \) and \( E_8 \) remain, in particular about the mod 2 invariants for \( E_7 \) and the mod 2 and mod 3 invariants for \( E_8 \); see, for instance, [Gar09, Open Problems 13.4 and 15.3]. A recent important discovery is a new mod 2 invariant of degree 5 by Nikita Semenov [Sem16]. A significant point is that his invariant is not defined on all of \( H^1(k, E_8) \), but only on the kernel of \( 15 \sigma_{E_8} \) (where \( \sigma_{E_8} \) is the Rost invariant for \( E_8 \), which has order 60), in the same spirit as the invariants \( e_n \) for quadratic forms are defined only on the kernel of some other invariant. This is an idea to keep in mind. We refer to [Gar16] for an outstanding overview about \( E_8 \), and in particular sections 8 and 9 for an overview about the cohomological invariants of \( E_8 \).

The goal of the project is to study cohomological invariants of structurable algebras. This is a class of algebras with involution, simultaneously generalizing Jordan algebras and associative algebras with involution. These algebras have been introduced by Bruce Allison in 1978 [All78], although the first non-trivial examples had already been constructed avant la lettre by Robert Brown in 1963 [Bro63]; these 56-dimensional algebras are now known as Brown algebras, and are related to groups of type \( E_7 \). Any structurable algebra \( \mathcal{A} \) gives rise to a Lie algebra \( K(\mathcal{A}) \) via the so-called Tits–Kantor–Koecher construction (TKK construction for short), and if \( \mathcal{A} \) is a central simple structurable \( k \)-algebra, then \( K(\mathcal{A}) \) is a simple Lie algebra. The connected component of its automorphism group, \( \text{Aut}(K(\mathcal{A}))^\circ \), is an adjoint simple linear algebraic group of positive \( k \)-rank; see [BDS17, Theorem 4.1.1]. It is believed that every such group arises in this fashion, but we are only aware of a proof of this fact for groups of \( k \)-rank 1 (in which case it arises from a structurable division algebra); see [BDS17, Theorem 4.3.1].

The definition of structurable algebras makes sense only for fields \( k \) of characteristic not equal to 2 or 3. The central simple structurable algebras for \( \text{char}(k) \neq 2, 3, 5 \) have been classified, first in characteristic 0 by Allison in [All78] (who mistakenly omitted one class) and then in general by Oleg Smirnov in [Smr90]. They fall into six (non-disjoint) classes:

1. Jordan algebras (in which case the involution of \( \mathcal{A} \) is trivial);
2. associative algebras with involution;
3. structurable algebras arising from hermitian forms;
4. structurable algebras of skew-dimension one [this includes the Brown algebras];
5. tensor products of two composition algebras or forms of such algebras;
6. a certain class of 35-dimensional algebras related to octonions [discovered by Smirnov].

For the study of exceptional groups, classes 3–6 are the most relevant. (Only the exceptional Jordan algebras, belonging to class 1, give another example of a structurable algebra giving rise to an exceptional group.) Of these three classes, the least understood is certainly class 4, namely the structurable algebras of skew-dimension one. For a very long time, it was only known that for each such algebra, there is a field extension $E/k$ of degree at most 2 such that $A \otimes_k E$ becomes isomorphic to a matrix structurable algebra, i.e., an algebra parametrized by $\begin{pmatrix} E & J \\ J & E \end{pmatrix}$ where $J$ is a cubic Jordan algebra over $E$, with a certain prescribed multiplication and involution, depending on some constant $\eta \in E$ [All90, Example 1.9]. Other than that, it was known that some (but not all) of these algebras can be obtained through a doubling process, the Cayley–Dickson process for structurable algebras starting from a Jordan norm of degree 4 [AF84].

Very recently, we discovered a uniform description of all possible structurable algebras of skew-dimension one in terms of (what we called) hermitian cubic norm structures over a quadratic étale extension $E/k$, or equivalently, cubic norm structures equipped with some selfadjoint semilinear autotopy [DeM17]. The resulting formulas for the involution and multiplication are surprisingly easy to write down; they are, in fact, easier than the earlier description of the matrix structurable algebras, and the case of a split étale extension (i.e. when $E = k \times k$) reduces precisely to these matrix structurable algebras.

Describe the objectives of the research.
Describe the envisaged research and the research hypothesis, why it is important to the field, what impact it could have, whether and how it is specifically unconventional and challenging.

The main theme of the project can be summarized as follows:

Study cohomological invariants of structurable algebras and relate them to cohomological invariants of exceptional linear algebraic groups.

We are aware of only two instances in the literature where cohomological invariants of certain structurable algebras have been considered. For Brown algebras, Skip Garibaldi showed in [Gar01, Remark 2.12] that there is a mod 3 invariant of degree 4, i.e., with values in $H^4(k, \mathbb{Z}/3\mathbb{Z})$. Since every simply connected split group of type $E_6$ is the automorphism group of a split Brown algebra, this invariant is in fact one of the invariants arising in [Gar09, Part II] mentioned above; see [Gar09, 11.7]. Already this specific case of Brown algebras is interesting and deserves a deeper investigation. In particular, Garibaldi only gives an explicit construction in the case where the Brown algebra is a matrix structurable algebra (called “of type 1” in his paper) and relies on descent theory to deduce that this invariant exists in general. It would be interesting to have an
explicit construction of this invariant for other Brown algebras ("of type 2"), in particular for the Brown division algebras, which are always of type 2.

The second instance is [BD13, Remark 7.7], where we indicated that for structurable algebras arising by the Cayley–Dickson process from a biquaternion algebra, there is a mod 2 degree 3 invariant that determines the structurable algebra uniquely up to isotopy, and this invariant coincides with the Arason invariant of a suitable 12-dimensional quadratic form. Notice that this is another instance of a structurable algebra of skew-dimension one. In [BD13], we were, in particular, interested in the connection with certain rank two forms of type $E_6$, $E_7$ and $E_8$ (where the case just mentioned corresponds to $E_8$), and it is worth investigating what this connection can imply with respect to the cohomological invariants.

Given the fact that already the investigation of the cohomological invariants of structurable algebras of class 1 (Jordan algebras) and class 2 (associative algebras with involution) have been active research areas for a long time, the investigation of invariants for all structurable algebras might seem very ambitious. On the other hand, the structurable algebras in classes 4–6 are of a very specific type and, in particular, of specific dimensions only, so their investigation is of a somewhat different nature. (Those of class 3, the structurable algebras arising from hermitian forms, are closely related to unitary groups, and the study of their cohomological invariants is expected to be essentially equivalent to the invariants of these hermitian forms.)

We list some specific goals we have in mind for each of these three classes separately.

- Study the cohomological invariants of the 35-dimensional algebras $T(𝒪)$ in class 6, and investigate the connection with groups of type $E_7$ (through the TKK construction). We believe that this is a very feasible task, since $T(𝒪)$ is completely determined by the octonion algebra $𝒪$ alone; thus it is very likely that we simply recover the unique non-trivial cohomological invariant for octonion algebras (namely the Arason invariant of its norm form). However, the connection with groups of type $E_7$ has an interesting feature. By [AF93], two such algebras $T(𝒪)$ and $T(𝒪')$ are isomorphic if and only if $𝒪 ≅ 𝒪'$, but on the other hand, they are always isotopic. In particular, the resulting group of type $E_7$ is always split, so the cohomological invariant "does not survive" when we pass to the group via the TKK construction. This raises another general question: what is the effect of the TKK construction on the cohomological invariants?

- Study the cohomological invariants of the tensor products $𝒜 = 𝒞_1 ⊗ 𝒞_2$ of two composition algebras, and of their forms, which are so-called twisted product algebras. Similarly to the well studied biquaternion algebras, such algebras have an associated Albert form, a quadratic form $q_{ Acrobat $ of dimension $dim 𝒞_1 + dim 𝒞_2 - 2$, and it is to be expected that (some of the) invariants of the algebra are related to (some of the) invariants of this quadratic form. It is worth mentioning that some very basic questions are still open. For instance, it is, to the best of our knowledge, not known in general whether $𝒜$ is a division algebra if and only if $q_{ Acrobat $ is anisotropic. (This is known to be true in
characteristic zero via indirect arguments using Lie algebras.) It is not unlikely that a good understanding of the cohomological invariants will help to settle this question.

- Study the cohomological invariants of the structurable algebras of skew-dimension one. Since we now know that every such algebra arises from a cubic norm structure together with a suitable semilinear automorphism, it is natural to ask whether we can construct invariants from these data. We believe that this class of algebras will provide the greatest challenge. One reason for this is that it is not clear at all what the connection is between different ways to represent the same structurable algebra in terms of these data, and this is of course essential if we want to construct well-defined invariants.

For each of these classes, we will, in addition, pay attention to the following aspects.

- For some of these structurable algebras, we will be able to find invariants by looking at the corresponding linear algebraic groups (for some suitable meaning of “corresponding” — which could be either via the TKK construction, or as their automorphism group, or yet something different!). A crucial aspect of our project will be to investigate whether these known invariants can be described directly in terms of the algebra, very much like the work of Petersson and Racine [PR95, PR96] for the cohomological invariants of groups of type $F_4$ in terms of the Albert algebras.

- A concept related to (certain) structurable algebras are so-called J-ternary algebras, introduced by Allison in [All76]; see also [ABG02, 3.12 and 6.61]. We have used J-ternary algebras to study certain rank 2 forms of groups of type $E_6$, $E_7$ and $E_8$ in [BD15]. More specifically, if $\mathcal{A} = C_1 \otimes C_2$ for two composition algebras $C_1$ and $C_2$, then there is a Jordan algebra $J$ “of reduced spin type” associated to the Albert quadratic form $q_{\mathcal{A}}$, and the algebra $\mathcal{A}$ has the structure of a J-module, making it into a J-ternary algebra. We will investigate the connections between these different algebraic structures and the corresponding rank 2 forms of type $E_6$, $E_7$ and $E_8$. (In this case, $C_1$ is an octonion algebra and $C_2$ is a composition algebra of dimension 2, 4 or 8, respectively.) Moreover, as pointed out in [BD15, Remark 3.4.3], there are two different structurable algebras around that play a role in the description of these forms, namely those just described, but also those arising from the Cayley–Dickson process from a tensor products of a quaternion algebra with a composition algebra of dimension 1, 2 or 4, respectively, and the link between these two algebras is not yet understood. Studying their cohomological invariants will certainly be helpful in this respect.

- Throughout the whole project, we should be attentive for possible cohomological invariants that are not defined on all structurable algebras of a given type, but perhaps only on algebras contained in the kernel of some other invariant, very much like the invariants $e_n$ for quadratic forms or like the Semenov invariant for $E_8$. 
Describe the methodology of your research.

Be as detailed as necessary for a clear understanding of what you propose. Describe the different envisaged steps in your research, including intermediate goals. Indicate how you will handle unforeseen circumstances, intermediate results and risks. Show where the proposed methodology is according to the state of the art and where it is novel. Enclose risks that might endanger reaching project objectives and the contingency plans to be put in place should risk occur.

Let us mention right away that for this research project, finding the correct methodology is already an important step towards the solution, and perhaps the description of the methodology will be less detailed than is to be expected. Nevertheless, we will mention some ideas that will get us started.

In conformity with the goals we have described, we will mention possible methods for studying each of the three classes of structurable algebras 4–6 separately.

- The algebras of class 6, of the form $T(O)$ for some octonion algebra $O$, are well understood. An elegant description of these algebras was found by Allison and Faulkner [AF93], who observed that they arise as the kernel of a “linear norm form” $\lambda$ from $S^2(O)$, the 36-dimensional subspace of symmetric elements in $O \otimes O$ with respect to the exchange involution $a \otimes b \mapsto b \otimes a$, to the base field $k$, determined by the rule $\lambda(a \otimes a) = N_O(a)$. This should allow us, hopefully without too much effort, to understand the cohomological invariants of $T(O)$. (Notice that by [AF93, Theorem 2.5], a form of $T(O)$ is always again isomorphic to some $T(O')$. ) It will also be interesting to find out whether there is a connection with the cohomological invariants of the algebra $O \otimes O$ itself (of class 5).

- The algebras of class 5 are the algebras $C_1 \otimes C_2$ for composition algebras $C_1$ and $C_2$ and forms of such algebras. Of course, if neither of these two composition algebras is octonion, then this is just an associative algebra with involution, so we will mainly be studying the case where $C_1$ is octonion. If $\mathcal{A}$ is a form of such an algebra which is itself not a tensor product of two composition algebras, then necessarily $C_1 \cong C_2$, and the resulting form is a twisted $(8,8)$-product algebra. (There is an explicit description of these algebras using a corestriction functor; see, for instance, [All88, section 2].) In all cases, the associated Albert quadratic form $q_{\mathcal{A}}$ ought to play an important role. In characteristic zero, it is known that for two such algebras $\mathcal{A}$ and $\mathcal{A}'$, the Albert forms $q_{\mathcal{A}}$ and $q_{\mathcal{A}}'$ are similar if and only if $\mathcal{A}$ and $\mathcal{A}'$ are isotopic. This gives a strong indication that the cohomological invariants of $\mathcal{A}$ are precisely the cohomological invariants of $q_{\mathcal{A}}$. Even so, we will explore to what extent these invariants can be obtained directly from the algebra. For instance, if $C_1$ and $C_2$ are both octonion, is there a connection between the Rost invariants of these octonion algebras and the cohomological invariants of $q_{\mathcal{A}}$? This is a question for which the answer seems “computable”, i.e., we can write down two arbitrary 3-Pfister forms (i.e., norm forms of octonion algebras), write down the corresponding Albert form, compute the invariants (they will certainly depend on the linkage number of the two norm forms), and find out what the connection is.
Structurable algebras of skew-dimension one (i.e., class 4) are essentially equivalent to *Freudenthal triple systems*. This is, in fact, an older notion; see, for instance, [Mey68]. This is also the terminology which is used in [Gar09, Section 12], where these triple systems are related to the internal Chevalley modules corresponding to the highest root for groups of type $D_4$, $F_4$, $E_6$, $E_7$ and $E_8$, giving rise to Freudenthal triple systems of dimensions 8, 14, 20, 32 and 56, respectively, and thus related to (hermitian) cubic norm structures of dimensions 3, 6, 9, 15 and 27, respectively. Even though we described the study of cohomological invariants of these structurable algebras as the most challenging case, it is at the same time the case where we seem to have a very wide array of tools at our disposal. We believe that also our new explicit description of all structurable algebras of skew-dimension one mentioned earlier [DeM17] will be invaluable in finding explicit descriptions of cohomological invariants.

The papers by Petersson and Racine [PR95, PR96], despite not being very recent, illustrate the spirit of the methods we have in mind. We will strive to find elementary descriptions of the cohomological invariants, but of course, we will not be ignorant for other successful approaches to study cohomological invariants.

**Provide a work plan, i.e. the different work packages and a detailed timetable.**

*Describe the different work packages (WP) the proposed research work will be divided in. Indicate for each WP the time that it is expected to take. You might use a table or another type of scheme to clarify the work plan.*

Not applicable. Unlike many other disciplines, mathematical research is not carried out by experiments that can be timed or divided into work packages. Instead, the workflow depends on the results that we will obtain, and might diverge into other directions than we have proposed; that is the nature of research in mathematics. On the other hand, we will of course be attentive to each of the different aspects we have outlined, and we will divide our time in such a way that each of these aspects will be investigated.

**Enumerate the bibliographical references that are relevant for your research proposal.**


Indicate below whether you think the results of the proposed research will be suitable to be communicated to a non-expert audience and how you would undertake such communication. FWO encourages its fellows to disseminate the results of their research widely, and valorize them where possible.

It seems unlikely that the results will be suitable to be communicated to a non-expert audience.
Short CV — Tom De Medts

Five key publications:


Earlier FWO grants:

- FWO travel grant (V 4/155B MW. 1493) for an extended research visit (4 months) in Bielefeld, 2007.

- Cosupervisor for the FWO research project “Dualities in discrete geometrical structures” (G.0086.06).

- Supervisor for the FWO research project “Moufang sets” (G.0140.09).

- Supervisor for the FWO research project “Automorphism groups of locally finite trees” (G.0110.12).